6000 1881

Mining District File Summary Sheet

DISTRICT	Rosebud
DIST_NO	4010
COUNTY If different from written on document	Pershing
TITLE If not obvious	Rosebud Project Modelling
AUTHOR	Clayton R; McIntosh W; Hendrickson R; Parrat R Muerhoff C; Knudsen H
DATE OF DOC(S)	1996
MULTI_DIST Y / N? Additional Dist_Nos:	
QUAD_NAME	Sulphur 7½'
P_M_C_NAME (mine, claim & company names)	Rosebud Mine; Rosebud Project; Hecla Mining Co Santa Fe Pacific Gold Corp.
COMMODITY If not obvious	gold; silver
NOTES	Correspondence; handwritten notes; articles on modeling and reserve estimation NOTE: Do not scan articles in appendices as they are copyrighted 35p.

Keep docs at about 250 pages if no oversized maps attached
(for every 1 oversized page (>11x17) with text reduce
the amount of pages by ~25)

SS: DD 8/1/08
Initials Date

DB:
Initials Date

SCANNED:
Initials Date

Revised: 1/22/08

ROSEBUD PROJECT MODELING

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6000 1881 4010

SANTA FE PACIFIC GOLD CORPORATION

RENO EXPLORATION OFFICE
250 SOUTH ROCK BLVD., SUITE 100
RENO, NEVADA 89502
TEL 702-858-8000 / FAX 702-858-8011

September 4, 1996

Ron Clayton

General Manager

Hecla Mining Company: Rosebud Project

501 S. Bridge Street

Winnemucca, Nevada 89455

RE: ROSEBUD PROJECT MODELING

Dear Ron Clayton:

The enclosed document covers my concerns with the current Hecla modeling process. My objective is to assure that the modeling procedure applied to the Rosebud Project will provide the best possible foundation for resource evaluation and mine planning. No criticism of your diligent staff is implied.

I would appreciate it if you could find the time to review the document and forward your comments to Robin Hendrickson or myself. We are available to answer any questions you may have.

At present this document is being distributed for comment to you, Ron Parrat, Robin Hendrickson and myself.

Santa Fe Pacific Gold Corporation

W. Skip McIntosh

W. Skip McIntosh

cc Robin Hendrickson

Ron Parrat

Files

SANTA FE PACIFIC GOLD
INTERNAL CORRESPONDENCE

DATE: SEPTEMBER 4, 1996

TO: Robin Hendrickson

FROM: W. Skip McIntosh

RE: Statistical Concerns RE the Hecla Rosebud Project
Block model in progress.

Background

During the course of recent discussions between myself and Charles Muerhoff, a couple is issues surfaced which may render validation of the current Rosebud block model (in progress) impossible.

I had assumed that the variography, gold domain zoning and model interpolation would be based on composite data. The previous block model, generated for Hecla by MDA using MedSystem, was based on composites. Discussions with Charles Muerhoff about modeling issues revealed that Hecla's new gold domain boundaries, variograms and the model interpolation were based on assay data. This modeling procedure may not pass an independent audit.

The use of assays as a basis for variography and interpolation as well as using variable block sizes in a model are at odds with standard modeling practice as I understand it. The reported Hecla modeling procedure is not compatible with the SFPG approach. The Hecla model would be difficult to defend in a 10K audit.

not relevant

Assays Vs Composites

The substitution of assay data for composites as a basis for variography and interpolation violates a basic step in SFPG's accepted modeling procedures. The use of composites as a basis for variography satisfies the normal geostatistical rule that samples should have the same support (i.e. length). Compositing to a uniform length removes the variable support which results from different assayed sample lengths. To paraphrase Pete Knudsen, 'the Variogram captures the spatial continuity of the mineralization, but the continuity is a function of the length of the sample' (in "A Short Course on Geostatistical Ore Reserve Estimation" by Peter Knudsen, Montana Tech ,1988, page 74 included as Appendix A and in Appendix C). Calculating a composite using assays of variable length will produce a variogram which represents a mix of variability due to length as well as variability due to spatial continuity.

The assay sample set is strongly clustered and includes very high grade outliers. Hecla has calculated semi variograms based on this data. Semi variograms are sensitive to high grade outliers therefore a relative variogram or correlogram may be more appropriate method to define spatial continuity.

not that many outliers on a zone by zone basis

Changing Block Size (Sub-Blocking)

Another problem relating to the statistical assumptions incorporated into the Hecla model in progress relates to the process of "Sub-blocking". This process was explained during the recent Surpac demonstration in the Hecla office in Winnemucca. The Surpac software allows both a preferred block size and a smaller block size limit for smallest allowable sub-block to be set. The software is constrained to define the largest possible block size but fits in smaller blocks against oblique planar surfaces such as gold domain boundaries. The result is that multiple sizes of blocks may be generated to most closely honor the cross cutting planar boundary with respect to volume. Grades are assigned to the centroids of the sub-blocks. Each sub block is given the grade of the full size block which is being sub-blocked. That is the interpolated grade for a block which has been sub-blocked will be the same value stored at each sub block centroid. Essentially every possible sub block has the same grade as the block being subdivided.

The assumption made in "sub-blocking" is that the block size can change without changing the variance. Typical geostatistical calculations indicate that simply changing the block size will not change the mean of the blocks but the variance of grades by block size is expected to decline as the block size increases.

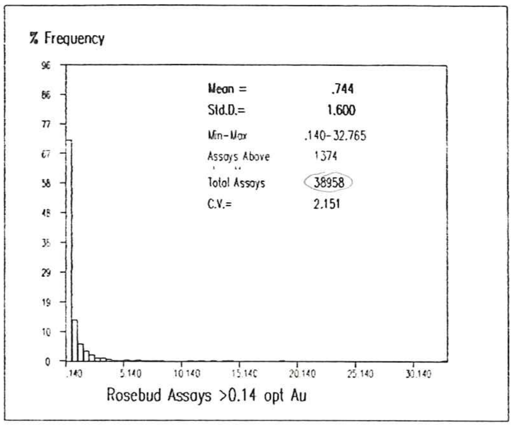
The assumption that a block can be defined with many sub-blocks each with the same grade ignores the necessary change in variance associated with this change in support. In order to define a smaller block in a block model, a new composite file should be built using the shorter length. The variogram of the new composites should be calculated and the smaller blocks interpolated . The new blocks would have the appropriate variance.

not relevant - sub-blocking may be used during mine planning not resource estimation.

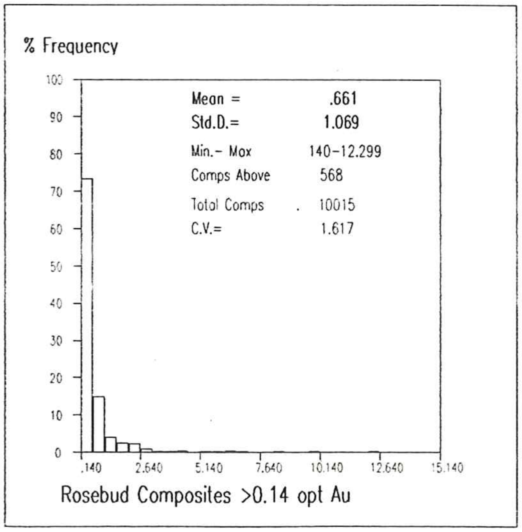
Change of Support

Both of the above issues are basically change of support issues. Mixing types of support (sample lengths.) and making changes to the block size without making adjustment for the difference in the variability are change of support problems. A geostastical description of change of support issues and volume variance can be found in Appendices B and C.

The concept of volume variance can be shown by comparing the distribution of values above a cutoff. The following charts show the change in univariant statistics between assays (smaller volume) and composites(larger volumes)



Assays greater than 0.14opt Au cutoff are 3.5 % of the total



? what is the composite length?

Composites greater than 0.14 opt Au are 5.7 % of the total

Selective mining units represent a much larger volume than the composites.

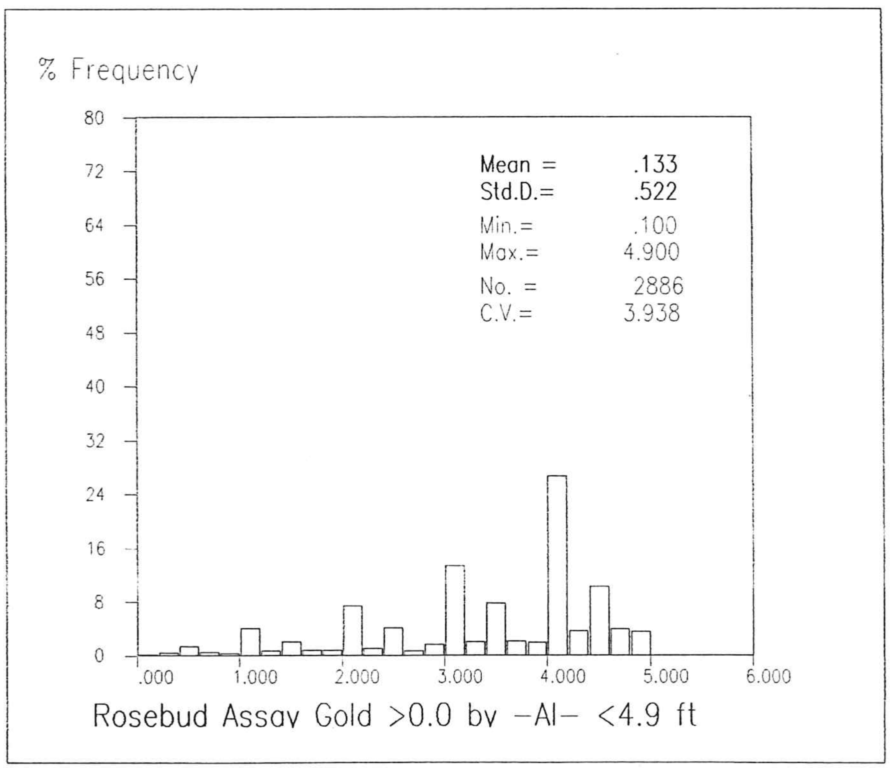
4

The distribution of selective miniing units above a cutoff value will follow the same trend as seen between assays and composites. The number of values above the cutoff will decrease as well as the mean grade , the maximum value and the variability compared to the composites.

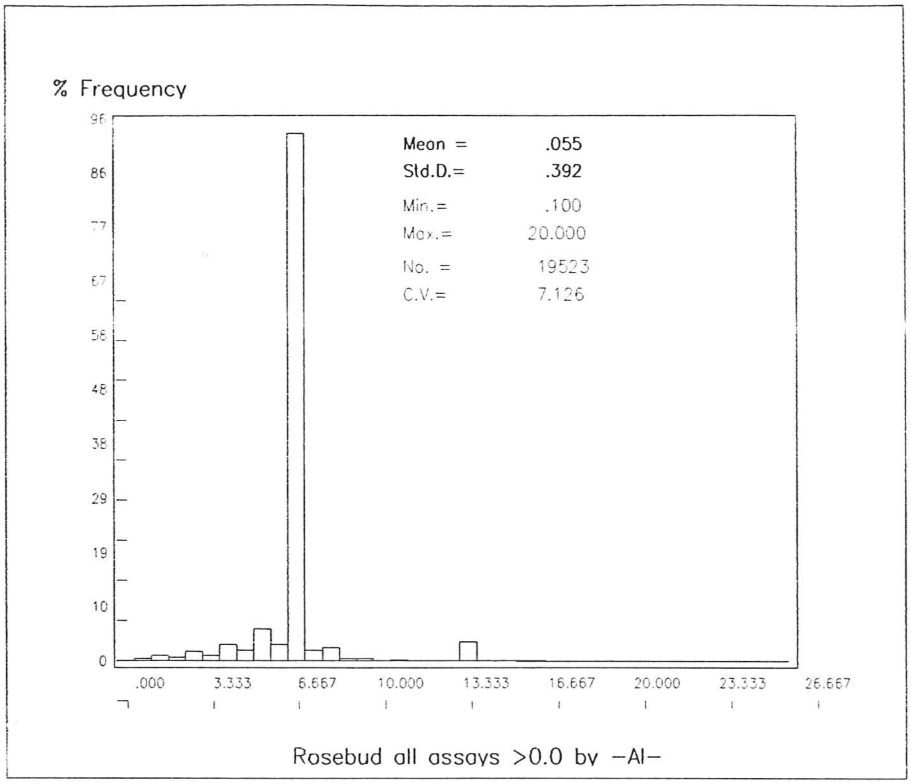
Assay Data

An examination of the grade distribution of assays by length was done by generating histogram plots of gold by length ranges. The charts of gold values between 0 and 4.9 feet, and the distribution of gold values in assays greater than or equal to 5 feet are shown below. A much higher mean grade is found in the short assays. Higher grade assays are associated with short sample lengths due to selective sampling of core samples. The result of weighting the short samples equally with the longer length samples will be a higher predicted grade in the block model.

This is done on a global basis - what is the effect on a zone by zone basis?



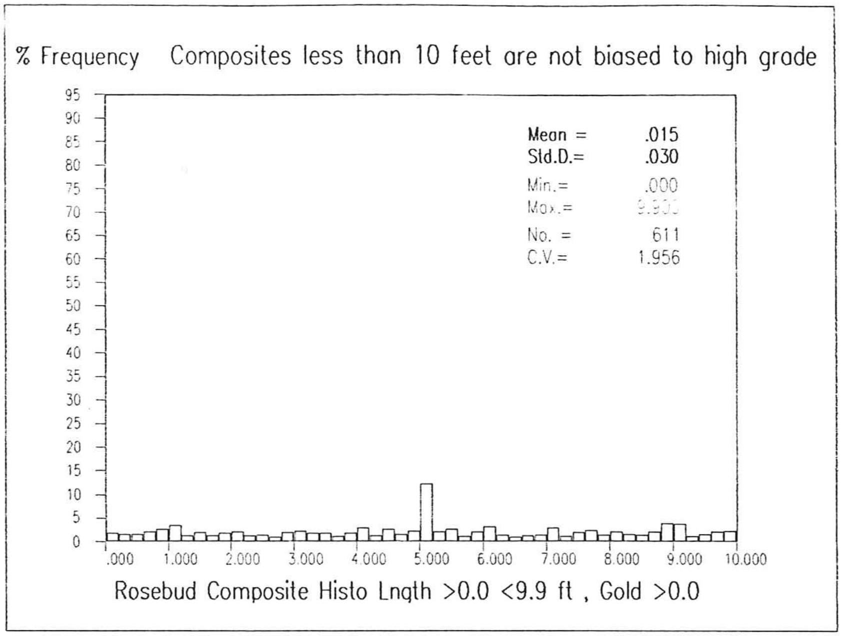
There is a bias to higher grades in shorter assay intervals as shown by the mean of the assays less than 5 feet compared to the mean of assays 5 feet and greater. The short assays make up 14.8% of the assay database.

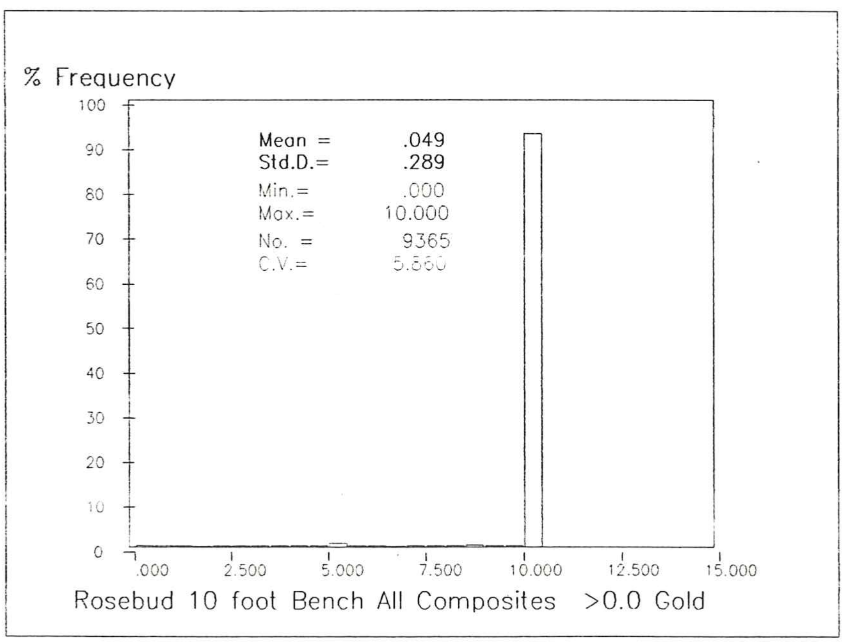


Shorter length composites result from irregular drill hole lengths in the first or last interval. This is an artifact of bench compositing which calculates the composite based on toe and crest elevations. Short composites make up 6% of the composite data. and can be reduced by using down hole compositing or applying length limits on the composites used in calculations. The influence of the small percentage of short composites can be minimized in the block modeling process by limiting the shortest composites used to 1/2 of the bench height (or 1/2 the Selective mining unit size).

throws away valid data

6





The graph shows that the mean of the 10 foot composites is nearly the same as the assay mean but with a lower Standard Deviation and lower C.V. (coefficient of variation)

duh

7

Conclusions and Recommendations

1) The modeling strategy currently being adopted (sub-blocking ,assay based variograms and assay based interpolation) for the Rosebud model will most likely lead to an overestimation of the grade and ounces when applying a cutoff grade for mining.

This is the problem which Lauren Roberts has described as being his experience at other operations i.e. the model has predicted more ounces than mining demonstrated. ?

The best way to eliminate the problems of changing support variograms and block sizes is to focus on "internal dilution" or the Volume Variance: change in support issues.

2) Composites should be used in the interpolation process. Grades should be estimated by Kriging, Inverse Distance power (power chosen by Michel David's method, Appendix D) and Nearest Neighbor methods. All three methods should be evaluated as a basis for mine planning.

if skippy had chosen to communicate w/ me he would know we are planning to try alternate estimation methods

3) The block size should be held constant at a size equal to the appropriate SMU for an underground mining project.

not relevant block size will be 10x10x10

4) Simplify the interpolation strategy minimum number of statistically supported mineral boundaries within 4 area domains instead of 19 different domains. Let the modeling define the grade distribution boundaries rather than trying to visually estimate them from the assays. There are inherent errors in the assays which could lead to bias in the grade zoning. ?

its the geology stupid!

any "errors" in assays would be incorporated into composites. what does "errors"? mean?

5) The highly clustered spatial distribution of drilling samples and the high grade outliers in the distribution of values in the South zone raises the problem of different local mean values within the densely drilled zones compared with the distal (lower grade zones). This condition would render the assumption of stationarity of the mean invalid. Hecla has not declustered the data before calculating variograms. The effect of clustering and non stationarity may be minimized by calculating the variogram using a method which reduces the effect of the local mean on the variogram curve. These methods include calculating a relative variogram, or using either the covariance or correlogram function to characterize the spatial continuity of the mineralization.

P.K. will comment

Kriging takes away the effects of clustering?

Appendices

Appendix A

Selections from "A Short Course On Geostatistical Ore Reserve Estimation" by H. Peter Knudsen, Division of Mining and Minerals Engineering, Montana Tech, Butte Montana May 1988.

Appendix B

Selections from " Lecture Notes: Short Course on Geostatistics prepared for Santa Fe Pacific Gold Corporation" by Ed Isaaks, Ed Isaaks & Co. 1670 S. Amphlett Blvd. Suite 306, San Mateo, California. June 10- 14, 1996.

Appendix C

Selections from "SME Mining Engineering Handbook" 2nd Edition Volume 1 Senior Editor Howard Hartman Published by Society of Mining, Metallurgy, and Exploration, Inc. Littleton, Colorado, 1992. Chapter 'Ore Reserve and Resource Estimation' by Alan Noble

Appendix D

"Dilution and Geostatistics". by Michel David in CIM Bulletin June 1988, pages 29 to 35.

9

Appendix A

Selections from "A Short Course On Geostatistical Ore Reserve Estimation" by H. Peter Knudsen, Division of Mining and Minerals Engineering, Montana Tech, Butte Montana May 1988.

A SHORT COURSE ON GEOSTATISTICAL ORE RESERVE ESTIMATION

by

H. Peter Knudsen

Division of Mining and Minerals Engineering
Montana Tech
Butte, Montana

May 1988

Validity of estimation

VARIOGRAM MODELING

The main purpose of a geostatistical study is to make a series of estimates of ore grade and tonnages. However, before these estimates can be made, a usable variogram must be obtained from the data. In this chapter, an outline of the general steps taken to get a variogram will be presented. The sequence of steps presented has two aims. First, before calculating a variogram, we need to answer several questions about the validity of using the variogram and kriging to make our ore reserve estimates. Second, the steps will be a guide to calculating and refining the variogram, so that we get maximum information from the variogram. The sequence of steps is the result of many trials and errors.

7.1 Assumptions

In Chapter 3, ordinary kriging was derived under the assumptions of second order stationarity of the underlying random function. Ordinary kriging is a valid tool even under the quite weak assumption of quasi-stationarity (the intrinsic hypothesis). This assumption says that within sliding neighborhoods, the first moment (the mean) is approximately constant and that the variogram exists. The assumption that the mean is approximately constant really means that there is no strong drift (or trend) in the data values. For instance, if a coal seam we were studying had a steady increase in thickness to the North, then this assumption would be violated and ordinary kriging might not be an appropriate tool for this deposit. Please note that all deposits will exhibit areas where there are local drifts or trends, but at some scale these deposits appear to have a constant mean. Drift is a scale phenomena. The second assumption concerns whether the variogram exists and is approximately constant across the region being studied.

No statistical tests can be used to determine whether or not a particular data set fits the above assumptions. In fact, the assumptions should be used not so much in a sense of determining whether the data fits the assumptions, but more in a sense of using the assumptions as a guide to define a data population that does fit the assumptions. Please note, in all statistical applications, the analyst defines the population to be studied. In an ore reserve study, the population being defined may be the whole deposit, or more often there will be several populations defined, based on the geology of the deposit.

7.2 General Steps in Making a Variogram Study

1. Stratify the area into (more or less) homogeneous zones. The purpose is to define populations that satisfy the assumptions given in Section 7.1. This can be done in many ways, but usually a thorough analysis of the geology and mineralization needs to be made and used as a guide in the definition of zones to be studied separately.

   A. Geologically different. Frequently, the mineralization in a deposit is strongly controlled by the rock type. For instance, at the Golden Sunlight Mine in Montana the gold occurs predominantly in a breccia pipe, but the surrounding precambrian sediments are also mineralized by to a much lesser degree and the spatial characteristics of the mineralization is much different in the sediments.

2. Determine what type of sample information is to be used.

   A. In most cases, we don't have the luxury to be choosy about what sample information is to be used in the study, because we frequently just don't have enough data. However, studies done on old mining districts will have a wide variety of sample information. In such cases, careful study may be necessary to determine if all the information is of the same quality and if different sample types should be grouped together. Assays from diamond drill core sometimes (maybe always) have different characteristics from rotary drill hole cuttings. & CAN data types be combined?

   B. Samples used in a study should have SAME SUPPORT (length)

   B. What length composite should be used? One of the important rules of geostatistics is that the samples used in a study should have the same support (i.e., length). Sample data that has irregular lengths should be composited to uniform length intervals. In deposits to be mined by open pit, the composites are usually chosen to be the height of the bench. Compositing serves more than one purpose. The variogram captures the spatial continuity of the mineralization. However, this continuity is a function of the length of the sample. Samples that are 1 inch long are not likely to be correlated over a distance of 100 feet, yet a composite that is 40 feet long may be correlated over a distance of several hundred feet.

3. Look at the data. There are many methods of data analysis and most of us have our own favorite methods. This step should actually be ongoing from the very beginning of any study, because it is extremely important to have a clean (without errors) data set. Errors can easily creep into any data set. There can be errors in the assay value due to mistakes in data entry, etc, errors in the coordinate information, and errors in the geological information of the sample.

   A. Maps and cross-sections. Besides their use in studying the geology of a deposit, they can also be used to check for errors in the data.

   B. Histograms and other statistical plots. Histograms and probability plots are extremely useful in looking at the data. The histogram of an assay distribution should look smooth and have a single mode. A bimodal distribution is usually indicative that we have mixed "apples and oranges." If we have defined our populations carefully, we shouldn't have a bimodal distribution. An example of this is shown in the histogram in Figure 7.2 of coal thicknesses in the Upper Freeport E seam in Pennsylvania. Here, the values above 65 appear to be out of place. This particular coal deposit is marked by a portion of the deposit that has three benches of coal that total about 70 inches thick and a portion that has only two benches of coal that average 48 inches thick. The thicker coal occurs along the margins of the swamp that formed the coal and clearly must be separately studied.

   The histogram should not exhibit a truncation of the data. In general we want to include a full range of the data from the area we are studying. We shouldn't arbitrarily remove all the data that is below cutoff.

Appendix B

Selections from "Lecture Notes: Short Course on Geostatistics prepared for Santa Fe Pacific Gold Corporation" by Ed Isaaks, Ed Isaaks & Co. 1670 S. Amphlett Blvd. Suite 306, San Mateo, California. June 10- 14, 1996.

Isaaks & Co.
Consultants in Spatial Statistics

— LECTURE NOTES —

Short Course on Geostatistics prepared for
Santa Fe Pacific Gold Corporation
June 10 -14, 1996

Bayshore Corporate Center 1670 S Amphlett Blvd Suite 306 San Mateo CA 94402 (415) 655-4607 Fax (415) 655-4608

ORE RESERVE MODELING

Contents:

- change of support
- change of support models
- implementing a change of support
- using grades zones or mineralization envelopes
- the estimation of mining reserves - grade control

CHANGE OF SUPPORT - practical importance

- Recovered quantities depend upon block size!

Ore Reserve Blocks
Actual Grades (moz/ton)

5	12	6
8	10.3	10.8
12.5	23	10.5

Selective Mining Units
Actual Grades (moz/ton)

2	1	4	3	5	2
6	11	18	23	9	8
9	19	22	8	23	11
2	2	6	5	4	5
8	22	19	26	18	9
9	11	28	19	9	6

Statistics (Ore Reserve Blocks)

Global	>10 moz/t
N = 9	N = 6
m = 10.9	= 67%T
	m = 13.2
	moz = 8.8T

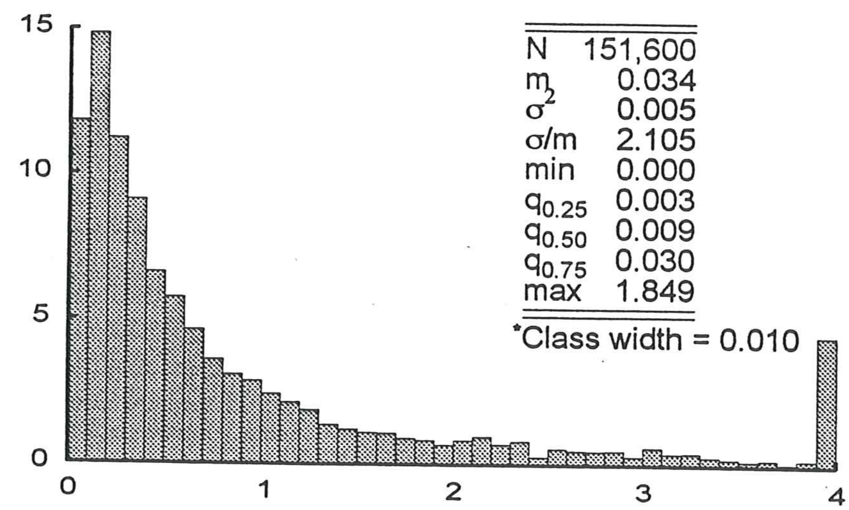
Statistics (Selective Mining Units)

Global	>10 moz/t
N = 36	N = 14
m = 10.9	= 39%T
	m = 19.3
	moz = 7.5T

CHANGE OF SUPPORT - practical importance

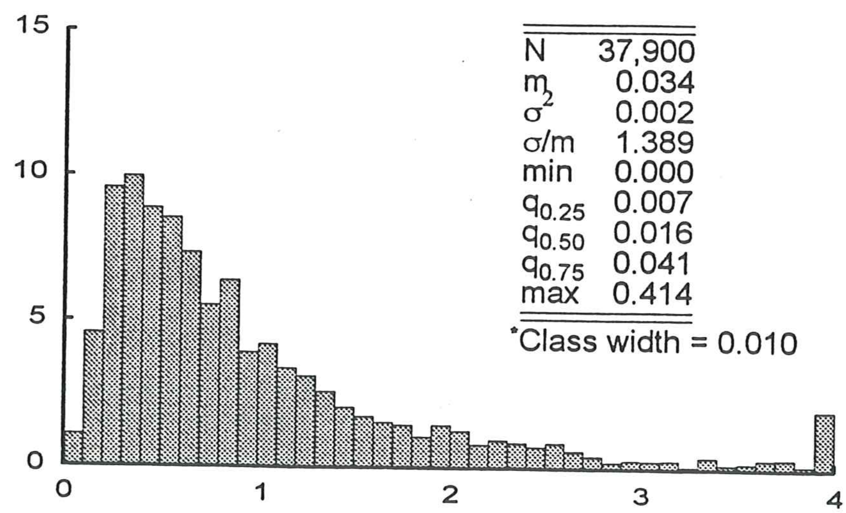
- the mean above a cutoff of 0.0 does not change with a change in support
- the variance of the block distribution decreases with larger support
- the shape of the distribution tends to become symmetrical as the support increases

Blast holes



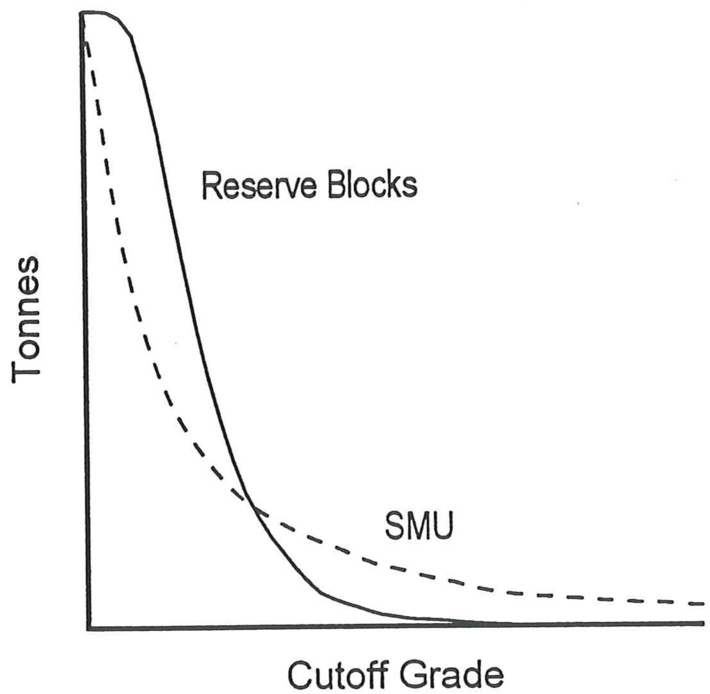
Central Limit Theorem

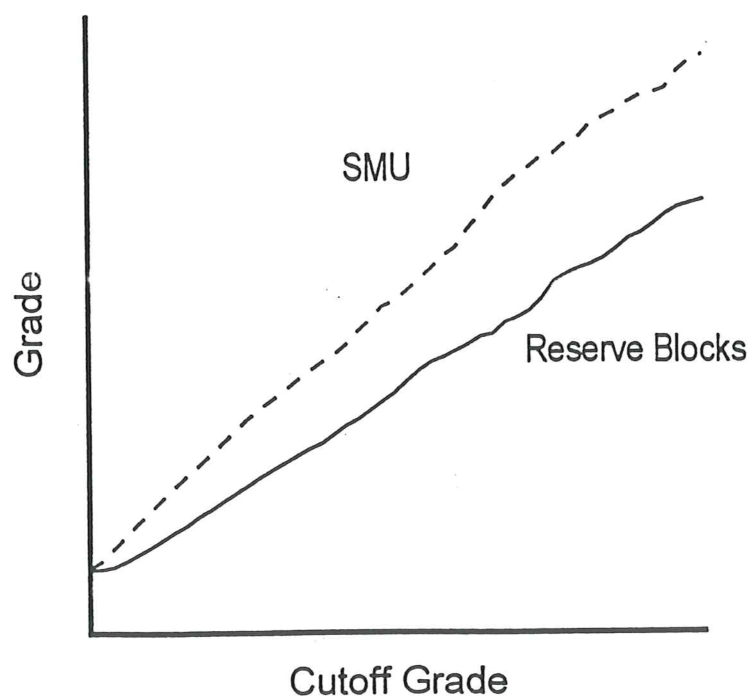
SMU



CHANGE OF SUPPORT - practical importance

- recovered quantities depend upon block size!





CHANGE OF SUPPORT - practical importance

- the ore reserve block model
  - block size typically linked to exploration drill hole spacing ⇒ relatively large blocks
  - estimated block grades are smoothed ⇒ resultant grade tonnage curves may not reflect actual grade tonnage curves of true but unknown block grades
- selective mining units
  - at the time of mining, the ore/waste selection or grade control procedure will result in grade/tonnage curves that are equivalent to those of blocks significantly smaller than the ore reserve blocks ⇒ selective mining unit (SMU)
  - the SMU or in situ mining recoveries may be quite different from the ore reserve block model predictions
  - these differences may be further exacerbated by misclassification at the time of mining
  - the SMU grade/tonnage curves plus the impact of misclassification provide the most accurate prediction of the material delivered to the mill

CHANGE OF SUPPORT - practical importance

- blast hole assays
  - often, attempts are made to reconcile mill production and ore reserve block model predictions to blast hole assays
  - however, the support of a blast hole assay is much smaller than that of a SMU or reserve block
  - thus, the predicted recoveries obtained from the blast hole assays may be very different from the SMU and/or ore reserve block model predictions

Conclusions:

- a method is required for making *a change of support* i.e., for calculating the relevant grade/tonnage curves for various block sizes

Standard error of the mean - to adjust nugget for different types of sampling support.

21

Appendix C

Selections from "SME Mining Engineering Handbook" 2nd Edition Volume 1 Senior Editor Howard Hartman Published by Society of Mining, Metallurgy, and Exploration, Inc. Littleton, Colorado, 1992. Chapter 'Ore Reserve and Resource Estimation' by Alan Noble

SME Mining Engineering Handbook

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Littleton, Colorado • 1992

persons. This is a standard procedure at many commercial data-entry shops that may dramatically reduce data-entry errors.

2. Manual comparison of a random sample of the original data sheets to a print-out of the database.

3. Scanning the data for outlier values. For example: drill locations outside the project limits, high and low assays, and sample intervals that overlap or are not continuous.

4. Comparison of computer-plotted data with manually plotted maps of the same data. Collar location maps and cross sections are especially useful to rapidly locate inconsistent collar locations and down-hole surveys.

Additional care and attention to detail and accuracy during data entry are essential. A database with a large number of errors may result in a resource estimate that is inaccurate and requires a complete revision to provide defendable results.

5.6.3 GEOLOGIC INTERPRETATION

The sample database represents a large three-dimensional array of point locations in a deposit. The sample data are quantitative and have been subjected to minimal reinterpretation after the original measurements. There is another body of geologic knowledge, however, that does not fit this description. This is the interpretation resulting from the geologist's assimilation of the large quantity of geologic data. These interpretative data are often represented on plan maps or cross sections that show outlines of the extent of geologic features or iso-grade contours that define ore zones. These interpretations combine to provide an interpretative geologic model that is one of the most critical factors in the resource estimation. Failure to develop an appropriate geologic ore body model is the most common reason for large errors in the resource estimates. As shown in Fig. 5.6.1, an inappropriate geologic model may lead to errors greater than an order of magnitude.

The geologist's interpretation of the ore body should be used as much as possible in developing the resource estimate. There are, however, practical limits to the amount of complexity that can be included in the resource model, and the geologic interpretation will be limited to critical inputs that define the shape and trends of the mineral zones at different cutoff grades and the character of the mineral zone contacts.

Examples of geologic features that are often modeled include

1. Receptive vs. nonreceptive host rocks.
2. Alteration types that accompany mineralization or create problems in beneficiation.
3. Faulting, folding, and other structural modifications.
4. Multiple phases of mineralization.
5. Post-mineral features such as oxidation and leaching.

Changes in lithology are often important variables in resource estimation because mineralization can vary due to physical or chemical attributes of the rocks. The differences may be distinct, such as the sharp contact between a skarn ore body and an unmineralized hornfels country rock. They also may be gradational, such as the gradual decrease in grade that is often observed between a favorable and slightly less favorable host in a porphyry copper deposit. Other important lithologic controls include barren post-mineral intrusive rocks, nonreceptive shale beds, and other unmineralized materials that are contained within the mineralized zone.

The effects of faulting will vary according to whether the faulting occurred before or after the mineralization, and to what processes accompanied the faulting. A simple post-ore displacement may create a discontinuity in the ore trends, preventing simple interpolation across the fault. The same type of fault occurring prior to mineralization may have little or no effect on the mineralization or may localize high-grade, vein-type mineralization that must be modeled independently of a more uniform disseminated ore body. It is also important to determine whether the fault is a thin, well-defined structure or many smaller structures in a complex, wide shear zone. In the first case, the fault is modeled as a simple surface with no thickness; in the second, the fault zone must be defined and modeled apart from the adjoining rock units.

Folding is particularly significant in sedimentary and stratabound deposits. Modeling of folding depends on whether folding happened before or after ore deposition, on the tendency of the ore zoning to follow the stratigraphy, on any remobilization that occurred with the folding, and on the creation of traps or other favorable structures. In addition to defining the shape of the folds, it is important to determine whether the mineralization follows the contours of the folds or is independent of the fold geometry.

Multiple phases of mineralization must be defined, particularly where they complicate the ore zoning pattern through overlapping, discordant trends, and through post-mineral oxidation or leaching. Secondary enrichment and oxidation will almost always require delineation of the modified ore zones.

The character of the ore zone contact must be determined and input into the resource model. A sharp contact will be handled as a discontinuity and the data used strictly independently on either side of the contact. A transitional contact, however, is a broad, gradational boundary that may require data selection from zones of tens of feet (meters) to over 100 ft (30 m) to achieve true differentiation between the different grade zones. As a transitional zone becomes thinner, it will eventually approach a sharp contact. For practical purposes, any transitional boundary thinner than the smallest selective mining unit will be modeled as a discontinuity.

In addition to definition of these physical ore controls and post-mineral modifications, a clear understanding of ore genesis will always be beneficial in creating a resource model. In the simplest case, the ore genesis will give clues to the behavior of the grade distributions and variograms; in other cases, the genetic structure is so dominant that it can be used as a direct control in the estimation of mineral resources.

5.6.4 COMPOSITING

Compositing is a procedure in which sample assay data are combined by computing a weighted average over longer intervals to provide a smaller number of data with greater length for use in developing the resource estimate. Compositing is usually a length-weighted average. If density is extremely variable (e.g., massive sulfides), however, compositing must be weighted by length times density (or specific gravity).

Some of the reasons for and benefits of compositing include

1. Irregular length assay samples must be composited to provide equal-sized data for geostatistical analysis.
2. Compositing reduces the number of data and may significantly reduce computational time, which is often proportional to the square of the number of data.
3. Compositing incorporates dilution such as that from mining constant height benches in an open-pit mine or from mining a minimum height/width in an underground mine.
4. Compositing reduces erratic variation due to a high nugget effect caused by erratic high-grade values.

There are several different methods for compositing that may be used depending on the nature of the mineralization and

24

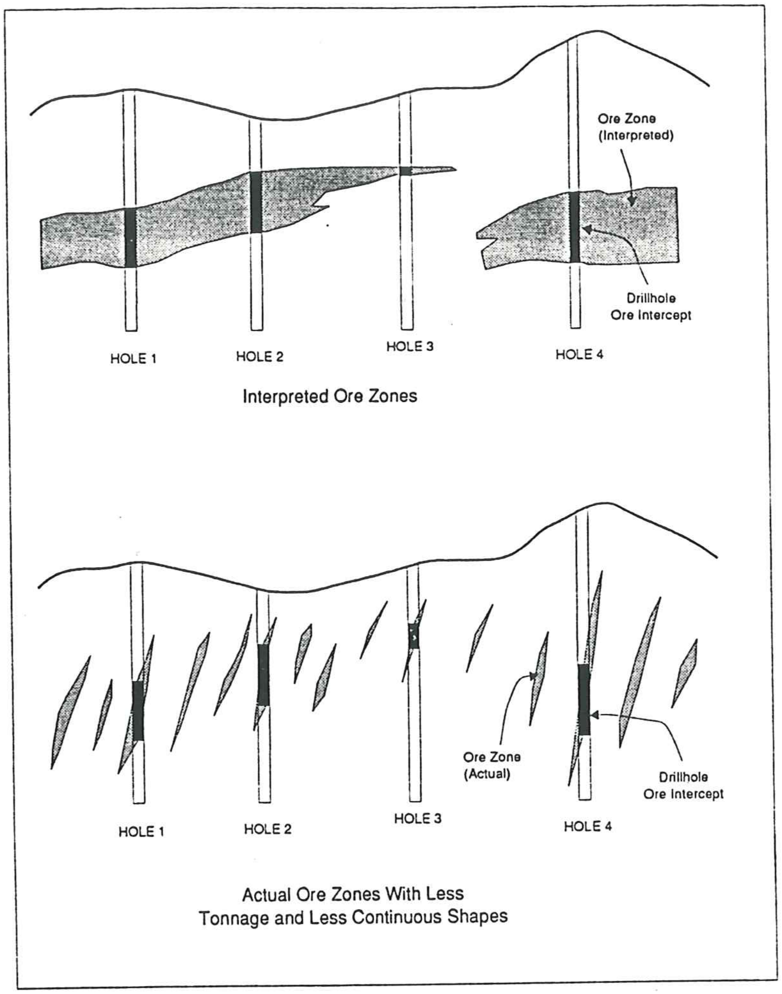

Fig. 5.6.1. Overestimation of ore reserves based on a geologic model that is less continuous than the actual ore zones.

the type of mining. Common compositing methods are (1) bench compositing, (2) constant length compositing, and (3) ore zone compositing.

Bench compositing is a method often used for resource modeling for open pit mining and is most useful for large, uniform deposits. Composite intervals for bench compositing are chosen at the crest and toe of the mining benches. Bench compositing has the advantage of providing constant elevation data that are simple to plot and interpret on plan maps. In addition, the dilution from mining a constant-height, constant-elevation bench is approximated by the bench composite.

Down-hole composites are computed using constant length intervals starting from the collar of the drillhole or the top of the first assayed interval. Down-hole composites are used when the holes are drilled at oblique angles (45° or less) to the mining benches, and bench composites would be excessively long. Down-hole composites should also be used when the length of the sample interval is greater than one-third the length of the composite interval to prevent overdilution when the sum of the lengths of the samples is much greater than the length of the composite.

Ore-zone compositing is a method of compositing that is used to prevent dilution of the composite when the width of the contact between waste and ore (or low grade and high grade) is less than the length of a composite. Use of bench compositing or down-hole compositing in this case may distort the grade distributions by adding low grade to the ore population and high grade to the waste population, resulting in underestimation of ore grade and overestimation of waste grades.

Ore-zone composites are computed by first identifying the interval containing each ore zone in the drillhole. Each ore zone is then composited individually as follows: (1) the length of the ore zone is divided by the desired length of the composite; (2) this ratio is rounded up and down to determine the number of composites that provide a length nearest the desired length when divided into the length of the ore zone; and (3) the ore zone is composited using length composites starting at the beginning of the ore zone and length as determined in the previous step.

A special case of ore-zone compositing is encountered in a vein or bedded deposit in that the width of the ore zone is determined by a combination of minimum mining thickness (height) and assay limits. In these situations, composites must be recomputed for each combination of assay cutoff grade and minimum mining thickness.

Geologic codes are usually assigned to composites according to the rock type, ore zone, or other geologic feature. This is often a simple procedure, since most composites will be computed from samples taken from a single geologic unit. Assignment of geologic codes to composites that cross geologic contacts is more complex, since the composite will be computed using data from multiple geologic units.

If the geologic contact is transitional and does not separate contrasting grade distributions, it is appropriate to assign the geologic codes according to the majority rule. If the composite

25

$$\sigma_{Xi,Xj} = \sigma_{S,D}^2 - \gamma(Xi,Xj) \quad (5.6.19)$$

$$\sigma_{B,Xi} = \sigma_{S,D}^2 - AVE(\gamma(B,Xi)) \quad (5.6.20)$$

$$\sigma_{B,D} = \sigma_{S,D}^2 - AVE(\gamma(B,B)) \quad (5.6.21)$$

where $\sigma_{S,D}^2$ is the variance of samples in the deposit, $\gamma(Xi,Xj)$ is the value of the variogram function between samples Xi and Xj, $AVE(\gamma(B,Xi))$ is the average value of the variogram between the block and sample Xi, and $AVE(\gamma(B,B))$ is the average value of the variogram between all points within the block.

Lognormal Kriging: *Lognormal kriging* is a method of nonlinear kriging that was developed to improve estimation when the underlying data are distributed according to a lognormal probability distribution. The basics of lognormal kriging include: (1) the variogram is computed using the natural logs of the data, (2) the kriging system is solved to provide a weighted average of the natural logs of the data, and (3) the kriged log average is then transformed back to normal values using lognormal transformation similar to that shown earlier in Eq. 5.6.1. The mathematics of lognormal kriging are complex and are discussed in Rendu (1978) and Journel (1978, 1980).

Complications in the practical application of lognormal kriging are many, including a strict requirement for a lognormal distribution and a variogram which is stationary over the field of estimation. Serious local and global biases may occur if either of these conditions are not met. In addition, there is a tendency for lognormal kriging to overestimate the high-grade end of the population when the coefficient of variation is greater than 2.0. Lognormal kriging is recommended only for special purposes where the results can be monitored closely and adjusted to prevent biases.

Indicator/Probability Kriging: *Indicator kriging and probability kriging* are related methods that are used to improve estimation when ore zones are erratic and grade distributions are highly variable and complex. Advantages of indicator kriging include less smoothing of estimated grades than ordinary kriging and robustness in handling nonstandard grade distributions.

The first step in indicator kriging is to set one or more cutoffs with which to define indicator variables. Given a cutoff g_c, the indicator variable is set to 1 if the grade is above g_c or 0 if the grade is below g_c (the order of the $\{1,0\}$ coding may be reversed); indicator variables are coded similarly for each desired cutoff. Variograms are modeled for each indicator variable and an expected value for each indicator is estimated using ordinary kriging and the appropriate indicator variogram.

The resulting indicator estimates, which may be interpreted as either the probability that the block will be above the cutoff or the percentage of the block that is above cutoff, are used to estimate the grade of the block as follows

$$g^* = \Sigma(I^*_j - I^*_{j+1}) \times g_j j = 0,1,2...n \quad (5.6.22)$$

where each I^*_j is the estimate for the indicator for cutoff j, g_j is the estimated grade for the interval j to $j+1$, and n is the number of indicator cutoffs. The interval grades g_j are usually estimated as the average of the cutoff grades for the interval, or, if the interval is large, may be estimated from the kriged grade of those data in the interval j to $j+1$. The prior method is more precise when a large number of indicator cutoffs are defined; the latter is most often used for a single cutoff.

Other Types of Kriging: Other types of kriging that are not widely used include universal kriging, cokriging, disjunctive kriging, and soft kriging. *Universal kriging* is a method to incorporate trends into the kriging equations. If the trends are defined according to a secondary variable, it is known as universal kriging with exogenic drift. A method of universal kriging that uses the geologists' interpretation of grade-zone trends as the exogenic drift is zoned kriging. *Cokriging* is the method of kriging that accounts for the correlation of a primary variable with a secondary variable, for example, gold with silver or molybdenum with copper, etc. When cokriging is used with qualitative secondary variables such as alteration, rock type, or other geologic features, it is known as *soft kriging*. *Disjunctive kriging* is a method used which attempts to estimate not only the local block grade but also the shape of the tonnage-grade distribution within the block.

5.6.8.6 Volume-variance Effects and Recovery Functions

The volume-variance effect refers to the inverse relationship between the distribution variance and the volume of blocks. The volume-variance effect is characterized by Krige's relationship as follows

$$\sigma^2{}_{B,D} = \sigma^2{}_{S,D} - \sigma^2{}_{S,B} \quad (5.6.23)$$

where $\sigma^2{}_{B,D}$ is the variance of blocks in the deposit, $S^2{}_{S,D}$ is the variance of samples in the deposit, and $S^2{}_{S,B}$ is the variance of samples in the block. The variance of samples in the block may be estimated from the variogram as follows

$$\sigma^2{}_{S,B} = \overline{\gamma}_{(S,B)} \quad (5.6.24)$$

where $\overline{\gamma}_{(S,B)}$ is the average of the variogram for samples within a block with the size and orientation of the mining block.

The volume-variance relationship is unimportant where the entire deposit is above the cutoff grade or where the ore is mined nonselectively. Generally, however, the cutoff grade is higher, and only a portion of the mineralized grade distribution is selectively mined as ore. The shape of the grade-tonnage distribution, as defined by the distribution variance, is then a critical factor in determining the grade and tonnage above cutoff.

For practical resource estimation purposes, the variance of mining blocks is generally larger than the variance of kriged resource estimation blocks. The variance of mining blocks is generally smaller than the variance of resource estimation blocks for polygonal estimation.

Polygonal estimation underestimates tons and overestimates grade for low cutoffs. At higher cutoffs, tonnage and grade are both overestimated. Kriging tends to overestimate tons and underestimate grade for low cutoffs. At higher cutoffs, tonnage and grade are both underestimated.

For polygonal estimation, the difference between estimated and mined reserves is usually handled with dilution factors where a fixed tonnage is added with a grade that is less than the cutoff. These dilution factors are adequate for correction of overall reserves but are not accurate for smaller areas if local grades vary significantly from the average grade. Caution must also be observed since dilution factors will vary according to the cutoff grade, the population variance, and the amount of variance reduction between the polygonal and mine block distributions. It should be noted that polygonal reserve estimates may require dilution factors for both volume-variance effects and contact-mining geometric effects.

Kriging reserves are corrected for volume-variance effects according to the distribution of mining blocks within the reserve block as (1) the variance and distribution of mining blocks within the reserve block is estimated, and (2) the tonnage and grade above cutoff is estimated for the block. The mining block distribution parameters are most effectively determined by compiling

26

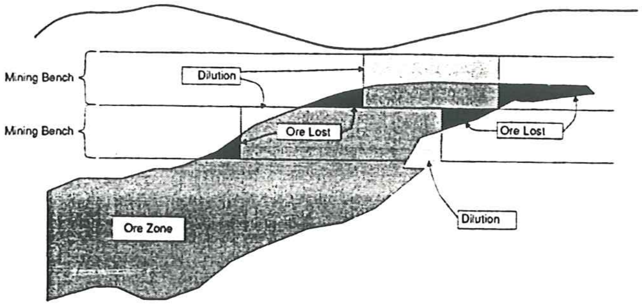

Fig. 5.6.19. Ore dilution and losses caused by mismatch between the mining geometry and the ore geometry.

production statistics of the grade-tonnage curves, or recovery curves, for several grade ranges of estimated blocks. Alternatively, a lognormal distribution may be assumed for mining blocks within reserve blocks. The variance of the distribution of mining blocks within reserve blocks may be estimated from production data. If production data are not available, the variance of mining block may be approximated by

$$\sigma^2_{b,B} = \sigma^2_{B,D} - \sigma^2_{S,B} + \sigma^2_{k,B} - \sigma^2_{k,b} \quad (5.6.25)$$

where $\sigma^2_{b,B}$ is the variance of mining blocks in the estimated reserve block, $\sigma^2_{B,D}$ is the variance of reserve blocks in the deposit, $\sigma^2_{S,B}$ is the variance of samples in the reserve block, $\sigma^2_{k,B}$ is the estimation (kriging) variance of reserve blocks, and $\sigma^2_{k,b}$ is the estimation variance of mining blocks (based on grade-control samples).

5.6.8.7 Dilution and Mining Losses

The estimated tonnage and grade must be adjusted for dilution of grade and losses of tonnage that occur in the course of mining. Dilution is waste that is not segregated from ore during mining, thus decreasing the grade of the ore and increasing the tons. Ore losses are due to the inability of the mining method to follow accurately and to segregate small isolated pods and small irregular offshoots from the main ore body. Dilution is most significant in deposits with sharp contacts between high-grade ore and barren waste and least significant in deposits with gradational contacts between ore and waste.

Dilution tonnage is estimated according to the quantity of waste mined with the ore based on the mismatch between ore body and mining geometry, overbreak in blasting, or lack of accurate location of the ore/waste contact as shown in Fig. 5.6.19. Care must be taken in estimating dilution that the actual ore/waste contact is not more irregular than the model since dilution will be underestimated as shown in Fig. 5.6.20. Dilution grade is estimated as the grade of the waste at the ore/waste contact. Mining losses and grades are estimated according to similar procedures.

5.6.8.8 Selection of Resource Estimation Methods

Selection of an appropriate resource estimation method depends on the geometry of the deposit, the variability of the grade distribution, the character of the ore boundaries, and the amount of time and money available to make the estimate. Deposit geometry determines the amount of detail that must be interpreted and input to the reserve estmation; the variability of the grade distribution determines the amount of smoothing that is required to estimate minable blocks; the character of the ore boundaries determines how grade will be estimated at the borders between different grade zones; and the available time and money determine the detail and effort that will be expended on the estimate. Considerations for selection of a resource estimation method are summarized in Table 5.6.1.

Cost: Simple, manual methods such as polygonal and cross-sectional estimations are the cheapest and quickest methods for estimation of resources when the quantity of data is small. This is usually the case for preliminary evaluations in exploration stages. As the number of data increase and a more detailed estimate is desired, computer-assisted methods should be used in order to save time and money. The least expensive computer-assisted methods are automated polygonal or nearest-neighbor methods and the most expensive methods involve extensive definition of geologic controls in conjunction with the more complex geostatistical methods. For further discussion of computer applications to ore estimation, see Chapter 8.4.

Ore Boundaries: The appropriate reserve and dilution estimation method is determined by the character of the ore/waste contacts. Sharp, simple boundaries are modeled with linear outlines defining discrete mineral zones. Individual estimations are made for each mineral zone; dilution is estimated based on the intersection between the shape of the mineral zones and the shape defined by the geometry of a mining method. A sharp, irregular boundary is also described with linear boundaries defining mineral zones; the actual ore-waste contact is much more irregular than the interpreted boundary, and dilution must be increased accordingly. Geometric methods are usually appropriate for ore bodies with sharp contacts, although kriging or inverse-distance methods may be used within the zones if supported by sufficient data.

Gradational boundaries are handled as transitional between different mineral zones; kriging or inverse-distance methods are most appropriate to model ore bodies with gradational contacts. Sufficient dilution for a gradational contact is usually incorporated in the modeling method.

Extremely erratic, irregular boundaries are difficult to define accurately and are most appropriately estimated using methods such as indicator kriging.

Deposit Geometry: *Simple geometry* is often found in tabular, stratabound deposits, veins, and structural zones. The geometry of these deposits is easily described using two-dimensional methods such as contouring of thickness and elevation. Few additional controls are required other than boundaries to limit the lateral extent of the mineral zones.

Deposits with *moderately complex geometry* include both deposits with simple geometry that have been moderately folded or faulted and deposits with large, simple, massive shapes such as porphyry copper and molybdenum. Definition of deposit geometry includes definition of fold axes, fault boundaries, and zoning of trends within the deposit. While these controls are not usually difficult to define, their definition is necessary to provide accurate resource estimates.

Deposits with *very complex geometry* are usually associated with structural deformation and are folded, faulted, stretched, and twisted to form extremely discontinuous shapes that are difficult to describe and model. Multiple ore controls such as a combination of stratigraphic and structural controls or multiple, overlapping pulses of mineralization also commonly form very complex shapes. Definition of deposit geometry requires detailed examination of structural geology and ore controls to provide cross sections or plan maps which define the shape and location of mineral zones. These sections or maps may then be used directly for manual resource estimation or may be digitized to provide control for a computer block model or three-dimensional

27

Appendix D

"Dilution and Geostatistics". by Michel David in CIM Bulletin June 1988, pages 29 to 35.

GEOSTATISTICS

Dilution and geostatistics

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ABSTRACT

The usual complaint of mines is that grade does not match exploration expectations. Quite often, more tons are mined than expected but at a lower grade. This can be seen as dilution. Geostatistics can help predict which dilution can be expected both in terms of grade and tonnage for different mining methods.

The usual vocabulary of geostatistics speaks of block distribution, block variance, size effect, information effect, all terms which say very little to the practitioner. In fact, all this can be rephrased in terms of dilution.

In this paper, the following is presented — an example which shows how reserves change with the size of selection units and number of samples taken, and tables which show the dilution that are bound to happen when dealing with the commonly encountered log-normal distribution. Finally, the paper shows how grade tonnage computations should be made in the case of an arbitrary distribution.

Introduction

Geostatistics progresses more and more rapidly if one judges by the number and quality of papers published in *Mathematical Geology* or at APCOM meetings. More and more case studies describing successful applications are published (David *et al.*, 1986) (Kwa and Mousset-Jones, 1987). It is estimated that more than 27% of the gold mines around the world use geostatistics. There is still, however, a lot of resistance from the remaining 73%. Geostatistics has more formulae and equations than it needs to reach year 2000. We will be well into the third millenium before all the models available now are actually used. The progress may come in the form of finding better ways to present known statistical results and getting away from the jargon of geostatisticians. One example may be in trying to explain how we can answer the most common plea of the mining industry: production grade is not what was expected in the feasibility study.

Year after year, the same distressing reports keep appearing. One does not usually notice, however, that tonnage is higher or that production lasts longer than expected. One is used to these facts and there is the term to describe it "dilution". Dilution is included in every reserve calculation, usually as a straight percentage which is added. For 40 years, geostatisticians have been addressing the problem and for at least 25 years they have claimed to have had it solved. Precise vocabulary has been created to describe the necessary concepts, but it is totally foreign to mining engineers or geologists. The concept of block variance is not something people manipulate in the field for instance. The author will try, in this paper, to show that dilution, due to grade variations and mining methods (not falling roof or caving walls), can easily be computed during the reserve calculation stage and thus can avoid costly surprises. It cannot eliminate surprises, but at least it can warn about what should be expected. A few tables, easily obtained, are presented to show the kind of dilution in grade and tonnage to be expected under a variety of conditions, as well as tables showing how the expected profit should be reduced because of dilution. It is then shown how all this is translated in statistical terms and finally a solution is shown for the case where grades are lognormally distributed and, as well, for an arbitrary distribution. This paper insists on principles. The mathematics have been presented many times; the essential is given in the appendix, and details can be found in David (1977).

A New Look at Dilution

A Simplified Definition of Dilution

There are many reasons for dilution which Elbrond (1986) calls ore losses. We will simply say that if we expect a grade x and we recover y, there is a loss of x-y, a relative loss of (x-y)/x; we will say the dilution is (x-y)/x. On tonnage, if we expect x and end up with y, the gain is y-x, or relatively (y-x)/x; we will call this the dilution on tonnage. These are the commonly used definitions.

M. David

Michel David is a graduate of l'École des Mines de Nancy, in France, where he specialized in applied mathematics and geostatistics; he received the degree of Ingenieur civil des mines. He received an M.Sc. in operations research from Université de Montréal in 1969 and a Ph.D. in 1973 from the same university.

Dr. David gained mining experience in coal, iron and gold mines in France, Canada and South Africa. He came to Montreal as a research associate in geostatistics at École Polytechnique and is currently professor in the Department of Mineral Engineering.

Dr. David is the author of over 70 publications and a textbook on geostatistical ore reserve estimation. He was a director of the N.A.T.O. Advanced Study Institutes in Rome and Tahoe and lectured at many universities in Europe, Canada, the United States, South America and Australia. He has been a consultant in geostatistics for the past 18 years to over 60 companies worldwide. Dr. David was a visiting professor at the Colorado School of Mines. From 1977 to 1981, he was the managing director of the Mineral Exploration Research Institute in Montreal and is currently president of Geostat Systems International.

Keywords: Geostatistics, Dilution, Grade control, Block distribution, Block variance, Ore reserve estimation.

Paper reviewed and approved for publication by the Geology Division of CIM.

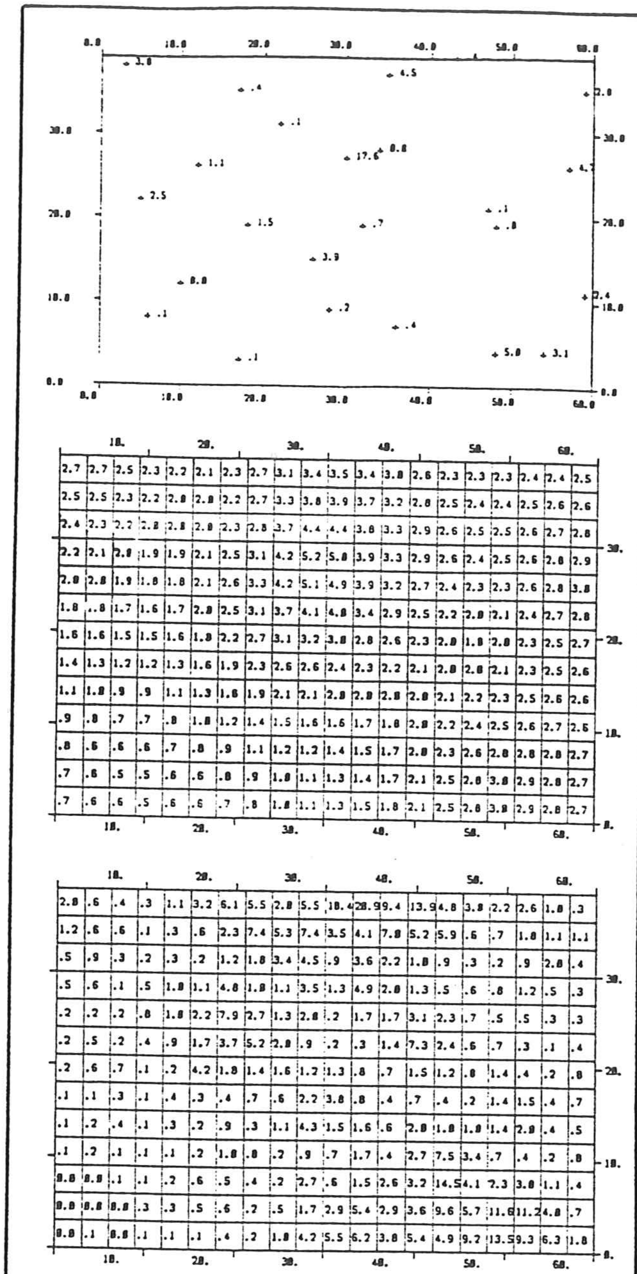

FIGURE 1. Sample values, real block values and estimated block values.

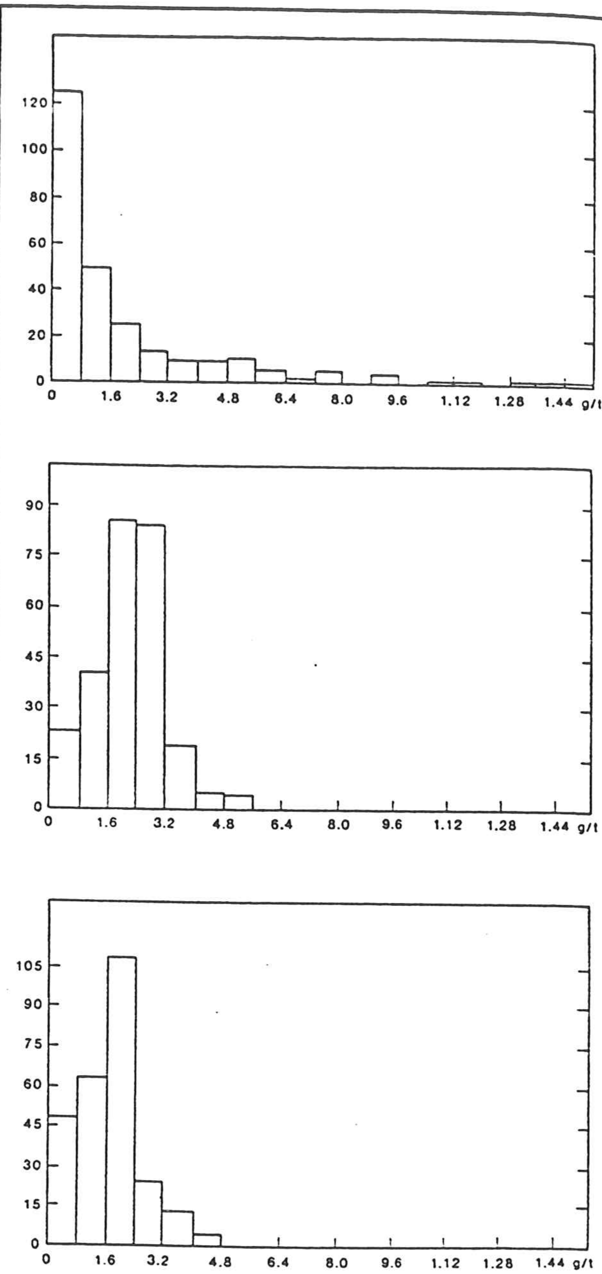

FIGURE 2. Distributions of sample grades, block grades and estimated block grades.

The Primary Cause for Dilution

Intuitively the first cause is the level of selectivity to be achieved at the time of mining, if mining with a teaspoon, better recovery is expected than with a 10-cubic-yard shovel. This is the "size effect", another reason is the degree of sampling. If a lot of samples are used it is less likely that waste will be sent to the mill and ore to the waste dump. This is the information effect, one more cause is the continuity of the ore. It is easier to do a good job in an iron mine than in a gold mine. A final parameter is the relative position of the cut-off with respect to the mean with no cut-off. High cut-offs are more difficult to follow.

Statistical Description of the Size and Information Effect

The grades known are the grades of samples, but the grades recovered are the real grades of blocks which have been selected on the basis of estimated values. As seen in Figures 1a, 1b and 1c, these are very different values. All these grades have different distributions (Figs. 2a, 2b, 2c). To characterize a distribution its variance or its square root is used, the standard deviation. It is known that the distribution is less variable as the size of blocks increases. In the common case of the lognormal distribution the dilution which should be expected in a number of circumstances will be computed. The results and how they can be obtained will be shown.

Results

It is now possible to compute the dilution involved if the calculations have been made on units (samples of variance σ^2 and in reality selection at the time mining is done on estimated values of blocks of variance σ_v^2. Three tables are shown here (Tables 1, 2, and 3) for different ratios of the cut-off with respect to the mean with no cut-off, 0.5, 0.75, 1.00. In gold this could be, for instance, 1 g/t with an average of 2 g/t, 1.5 g/t with an average of 2 g/t and 1.5 g/t with an average of 1.5 g/t. The tables are presented as a function of σ/m the coefficient of variation of the samples, which is a common measure of the variability of grades, vs the coefficient of variation of the selection units which will always be smaller than that of samples. The first table

TABLE 1. Ratio of cut of grade to average grade: 0.5

σ_0/m \ σ_v/m	0.50		0.75		1.00		1.25		1.50		1.75		2.00		2.25		2.50	
0.50	0	0																
0.75	-11	17	0	0														
1.00	-21	35	-11	15	0	0												
1.25	-28	51	-20	28	-10	12	0	0										
1.50	-34	66	-26	41	-17	23	-9	10	0	0								
1.75	-39	79	-32	53	-24	33	-15	19	-7	8	0	0						
2.00	-43	92	-36	63	-29	42	-21	27	-14	16	-7	7	0	0				
2.25	-37	103	-40	73	-33	52	-26	35	-19	23	-12	13	-6	6	0	0		
2.50	-49	114	-43	82	-36	59	-29	42	-23	29	-17	19	-11	12	-5	5	0	0

TABLE 2. Ratio of cut of grade to average grade: 0.75

σ_0/m \ σ_v/m	0.50		0.75		1.00		1.25		1.50		1.75		2.00		2.25		2.50	
0.50	0	0																
0.75	-14	20	0	0														
1.00	-25	37	-12	14	0	0												
1.25	-33	52	-22	27	-11	11	0	0										
1.50	-39	65	-29	38	-19	21	-9	9	0	0								
1.75	-43	78	-34	48	-25	30	-16	17	-8	7	0	0						
2.00	-47	89	-39	58	-30	38	-22	24	-14	14	-7	6	0	0				
2.25	-50	99	-42	66	-34	46	-27	31	-19	21	-12	12	-6	6	0	0		
2.50	-53	109	-45	74	-38	53	-31	38	-24	26	-17	18	-11	11	-5	5	0	0

TABLE 3. Ratio of cut of grade to average grade: 1

σ_0/m \ σ_v/m	0.50		0.75		1.00		1.25		1.50		1.75		2.00		2.25		2.50	
0.50	0	0																
0.75	-15	10	0	0														
1.00	-25	20	-13	9	0	0												
1.25	-33	30	-22	18	-11	8	0	0										
1.50	-39	38	-29	26	-19	15	-9	7	0	0								
1.75	-44	47	-35	33	-25	22	-16	13	-8	6	0	0						
2.00	-48	55	-39	40	-30	29	-22	19	-14	12	-7	5	0	0				
2.25	-51	62	-43	47	-35	35	-27	25	-19	17	-12	10	-6	5	0	0		
2.50	-54	69	-46	53	-38	41	-31	30	-24	22	-17	15	-11	9	-5	4	0	0

TABLE 4. Ratio of cut of grade to average grade: 0.5

σ_0/m \ σ_v/m	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50
0.50	0								
0.75	-5	0							
1.00	-9	-5	0						
1.25	-13	-9	-4	0					
1.50	-17	-13	-8	-4	0				
1.75	-19	-15	-11	-7	-3	0			
2.00	-21	-17	-13	-9	-6	-3	0		
2.25	-23	-19	-15	-11	-8	-5	-2	0	
2.50	-25	-21	-17	-13	-9	-7	-4	-2	0

considers a cut-off of half the mean ($x_c/m = 0.5$). If the coefficient of variation is 1 for samples, and selection is made on units the coefficient of variation of which is 0.5, Table 1 says that the dilution (loss) on grade is 21% and on tonnage (gain) 35%. There is a simple relationship between the coefficient of variation σ/m and the logarithmic standard deviation of the log-normal distribution ($\sigma/m = e^{\beta 2}-1$). Details of the calculations are given in Appendix 1.

Results shown by these tables are quite frightening. Coefficients of variation of samples (σ/m) in gold are typically above 1.5, sometimes 2 or more. One sees that for this case even a slight reduction in the variance of blocks, easily means dilution of more than 20%.

As an example, take the case of a typical Abitibi gold deposit. The coefficient of variation of samples may be 1.5, and the coefficient of variation of blocks as small as 2 m by 2 m by 2 m may be 1. Assuming a cut-off of 1 g/t and an average of 2 g/t, it means after Table 1 a dilution of 17% on grade and an increase in tons of 23%. One can, of course, compute the lost profit (Tables 4, 5 and 6). The profit is simply measured by the tonnage recovered multiplied by the difference between the mined grade and the cut-off grade multiplied by the metal price p.

$$P_v(x_c) = (m_v(x_c) - x_c).p$$

The expected profit was from sample values:

$$P(x_c) = (m(x_c) - x_c).T(x_c).p$$

The relative loss due to dilution is then

$$[P_v(x_c) - P(x_c)] \;/\; P(x_c)$$

It is interesting to note that it is not a function of the average of

TABLE 5. Ratio of cut of grade to average grade: 0.75

σ_o/m \ σ_v/m	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50
0.50	0								
0.75	-15	0							
1.00	-25	-12	0						
1.25	-32	-19	-9	0					
1.50	-36	-25	-15	-7	0				
1.75	-40	-29	-19	-12	-5	0			
2.00	-42	-32	-23	-16	-9	-4	0		
2.25	-44	-34	-26	-19	-13	-8	-4	0	
2.50	-46	-36	-28	-21	-15	-10	-6	-3	0

TABLE 6. Ratio of cut of grade to average grade: 1

σ_o/m \ σ_v/m	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50
0.50	0								
0.75	-29	0							
1.00	-42	-19	0						
1.25	-50	-30	-13	0					
1.50	-55	-37	-22	-10	0				
1.75	-58	-41	-28	-17	-7	0			
2.00	-61	-45	-32	-21	-13	-6	0		
2.25	-62	-47	-35	-25	-17	-10	-5	0	
2.50	-64	-50	-38	-28	-20	-14	-9	-4	0

the deposit, but only of the relative value of the cut-off to the average with no cut-off and the variability of the units. In this example, (which is a very conservative example), it would be 8%. If the average with no cut-off is only 1.5, the loss becomes 15%. Tables 4, 5 and 6 show that the higher the cut-off with respect to the mean, the bigger the loss.

What Makes the Variance Change?

The Size Effect

The variance of samples is made of the nugget effect, plus a term due to the continuity of the ore. The minute blocks rather than samples are considered, the nugget effect disappears. Intuitively, if a block of one cubic foot is moved a fraction of an inch, its grade will not change.

The variances of blocks is given by the variogram of grades according to well known formulae (David, 1977). Charts and now programs on microcomputer are available to compute it. In an open pit context, it is fairly easy to know the intended size of selection units, underground there is no such thing as sending a car load to the mill or the waste dump, but one stops a stope at a certain point, going up or in the walls. The size of selection units is not easy to define, but at least one is sure it is a block, not something as small as a sample. Consequently, the variance is at least reduced by the nugget effect which, in gold deposits, may represent up to 50% of the variance, sometimes more. A similar situation occurs in Saskatchewan uranium deposits where selection unit is a car load. Base metals deposits show smaller nugget effect and, consequently, show fewer surprises. Now, selection can never be made on real values, only on estimated values.

The Information Effect

Selection can only be made on estimated values, David (1977), Journel and Huijbregts (1978). If there are only a few samples, the information will be poor and many mistakes will be made. If more samples are taken, information will be better and fewer errors will be made. Thus, the variance to be considered is not the variance of real values. Neither is it the variance of the presently estimated values. At the time of mining, more information will be available. What is needed is the variance of future estimated values. This can be obtained ahead of mining using the smoothing relationship of geostatistics (David, 1972).

The variance of estimated grades $\sigma^2_{v^*}$ is equal to the variance of real grades σ^2_v less the estimation variance σ^2_k (kriging variance)

$$\sigma^2_{v^*} = \sigma^2_v - \sigma^2_k$$

Intuitively, of course, it is known that fewer mistakes will be made if a better estimation is made. A better estimation can be achieved two ways, with more samples or with a better estimation method or both. From this it can also be seen that it is possible to calculate the expected benefit of spending more money on sampling. It is fairly easy to show that in many cases, more money spent on sampling means a better selectivity and, consequently, a better profit.

Recovery with a Cut-off

Case of the Lognormal Distribution

When applying a cut-off x_c to a certain size mining unit, one will recover a proportion of the total tonnage at a certain grade. In the case of a lognormal distribution, simple formulae are available to predict the expected tonnage and grade recovered. These formulae are a function of β^2, the logarithmic variance of the selection units and of x_c/m_o where m_o is the average with no cut-off. One can compute recovery curves as a function of β^2. Such curves are shown in Figure 3. As they are a function of (x_c/m_o), there is one set of curves for each value of x_c/m_o or for a given deposit for each value of x_c. One can see that depending on the value of x_c, the sensitivity to β^2 may vary considerably. In other words, in some deposits, selectivity may pay, in others it will not.

An Example in a Tungsten Deposit

For example, look at the case of a tungsten deposit in a granite which is to be mined by open pit and where one questions the desirability of selectivity. The logarithmic variance of blocks as a function of their size is shown in Figure 4. One can see that changes are important for very small blocks. This kind of curve can be easily obtained with interactive geostatistical programs.

Now, the impact of sampling can be seen in Figure 5 which shows the logarithmic estimation variance associated to different block sizes and different sampling patterns, when blocks are estimated in the optimum manner, i.e. by kriging. These results are obtained in a few minutes using the KRIVAR program (GSII, 1982), which only requires the variogram parameters, the size of the block to be estimated and the size of the sampling grid.

From Figure 5, it can seen that the best to be done with a grid of 3 m to 4 m is the reduce the variance by about 0.045. Then Figure 3 shows how recovery varies with β^2, using the

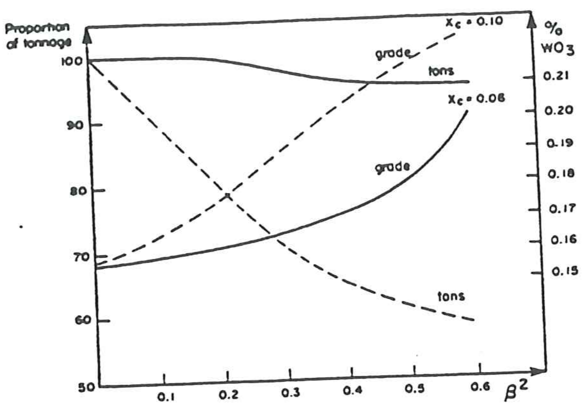

FIGURE 3. Grade-tonnage curve as a function of ß².

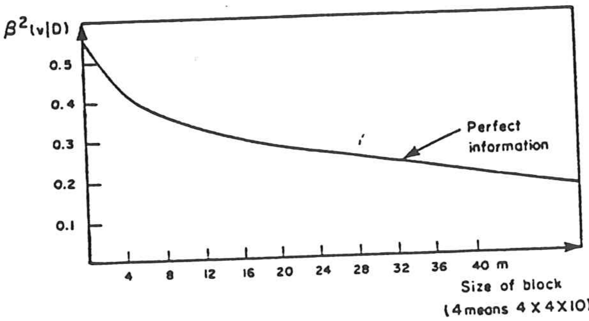

FIGURE 4. Logarithmic variance of blocks as a function of their size.

value read from Figure 4 diminished by 0.045. It is easy to make a sensitivity analysis taking into account the cost of sampling vs the expected quantity of metal to be recovered.

Case of an Arbitrary Distribution

Despite the fact that the lognormal distribution is very frequent, it is not universal. Geostatistics has techniques to compute the distribution of blocks in any case. The most common and simple method is called, in the jargon, the "affine" transformation. Variograms give the variance of the block distribution, but it does not give its shape. The affine correction assumes that the shape of the distribution remains the same. Strictly speaking, it is assumed that the cumulative frequencies of the standardized sample grade and the standardized block grades are the same:

$$F\frac{(x-m)}{\sigma} = F\frac{(x_v-m)}{\sigma_v}$$

This means that the block grade for which the frequency is the same as for sample grade x is

$$m + \frac{\sigma_v\,(x-m)}{\sigma}$$

This way, the distribution of block grades can be computed point by point.

An example in a Saskatchewan uranium deposit gave the results which can be seen in Figure 6. Again, these results can be considered as quite dramatic. It must be understood that grades which are seen on a plan or section are sample grades, they are real, yes, but blocks as small as a sample would not be mined, and even if they were, their real grades would never be known.

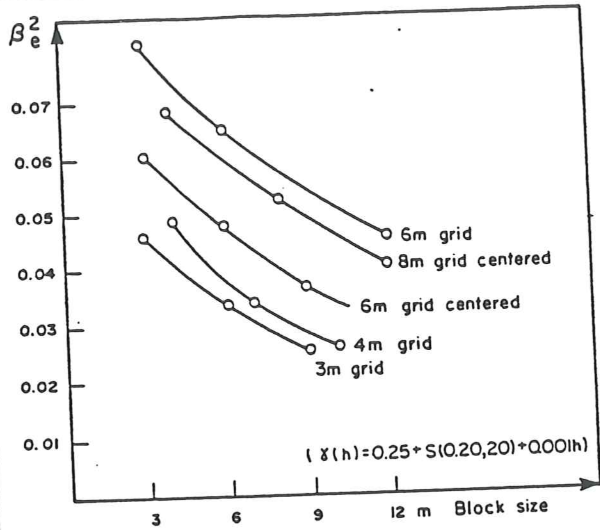

FIGURE 5. Estimation variance as a function of block size and sampling density.

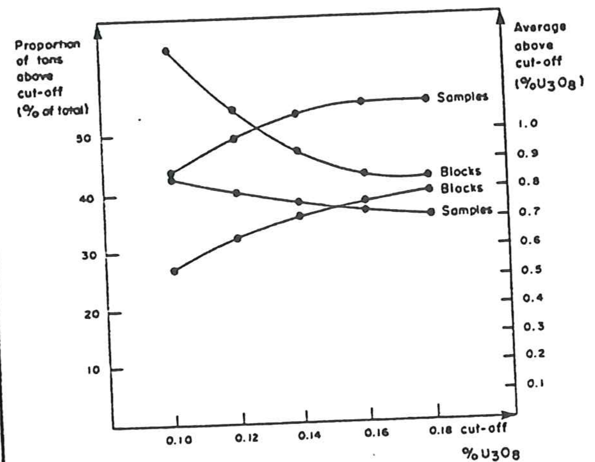

FIGURE 6. Tonnage and average recovered grade as a function of the size of selection units.

Improving Grade Control by Reducing Dilution with Better Samples

In open pit gold mines of Western Australia, a frequent method of grade control is to collect channel samples every metre on sampling lines cut 5 m or 10 m apart. It is clear, of course, that when more samples are collected there is better discrimination between ore and waste. Increasing the number of samples will improve the quality of the estimated values on which a decision will be made. The small mining units are 2.5 m by 1 m. What must be established is the real distribution of these units to obtain the best possible case, and the distribution of estimated values obtained from kriging using different possible sampling grids. Again, it will be considered that the distributions are lognormal and, from the usual formulae, we will be able to obtain the tonnage and grade recovered for each possible sampling pattern. According to the changes in recovered tons and grade, measured in dollars, a decision can then be made as to which sampling pattern is most suitable.

Maximum Possible Recovery

It is easily established from the variogram of samples that real grades of small blocks have a logarithmic variance of 0.45. The average in the area considered is 1.66 g/t hence, using the

TABLE 7. Per cent tonnage recovered and average grade

Pattern	$T(x_c)$	$m(x_c)$
3 samples lines 5 m apart	55.5	2.03
9 samples lines 2.5 m apart	52.0	2.15
6 samples lines 2.5 m apart	48.78	2.29
ideal	42.7	2.67

usual recovery formulae for the lognormal distribution, it can be found that for a cut-off of 1.5 g/t, the maximum recovery would be 42% of the tonnage at 2.7 g/t.

The Information Effect, Dilution Due to Sampling

Starting with a "full" sampling pattern (Fig. 7), making use of 9 samples on 3 lines to estimate a block and using the KRIVAR, (GSII, 1982) program, for instance, a kriging variance of 0.288 is obtained. Now to obtain the variance of the estimated values, $\sigma^2_{z^*}$, the following smoothing relationship is used:

$$\sigma^2_{z^*} = \sigma^2_z - \sigma^2_k = 0.45 - 0.288 = 0.162$$

Using the same recovery formulae, but with the new variance (this will give a recovery of 52% of the tonnage at 2.15 g/t) one can see the dilution as could have been expected 42% of the tons at 2.7 g/t.

Reducing the Number of Samples in the Estimation

Instead of using 9 samples, one can try to use only 5 (Fig. 8a) — the kriging variance stays the same at 0.288. If it is attempted to reduce the number of samples to 3 (Fig. 8b) the kriging variance is increased to 0.36 — the variance of estimated values is reduced to 0.11. Recovery would be 55.5% of tons at 2.03 g/t, or a further dilution. It is not a very good idea not to use all the samples available in the kriging program.

Increasing the Number of Samples Collected

Suppose now that the line density is increased to 1 every 2.5 m. Using a pattern with 6 samples we now have a kriging variance of 0.218 or a variance of estimated values of 0.232.

From these new variances, the distribution of estimated values can be established and the new grade tonnage curve compared with what one would have in the ideal case. Looking at a cut-off of 1.5 g/t for instance, we have the results of Table 7. One can see the dilution with a smaller number of samples.

Decreasing the Number of Samples Collected

Let us now suppose that only every second sample is assayed. Then a block can be in one of two situations with respect to the samples (Figs. 9a and 9b). It turns out that the kriging variance is virtually the same in both cases, 0.332 in one case and 0.337 in the other. By taking an average of 0.335 it can be seen what this does to the recovery. The variance of estimated values now becomes 0.45 to 0.335 = 0.115. Recovery is now 55.1% at 2.04 g/t.

Choosing a Sampling Pattern

All that remains to be done is the calculation of the expected profit corresponding to each case. This is, of course, very easily achieved. In conclusion, it can be seen that the geostatistical theory gives the necessary tools to decide on a sampling pattern. All that was needed in this case is a program to compute variograms, a program to perform kriging and a program to compute

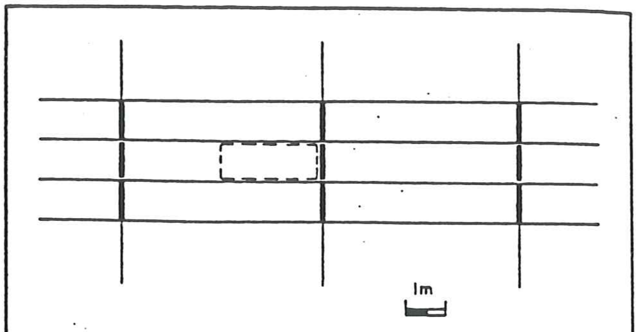

FIGURE 7. Full sampling pattern.

the lognormal recovery, in addition to remembering the smoothing relationship.

Conclusion

From the three examples discussed it can be seen that big surprises can be avoided with a minimum of effort. The original jargon of geostatistics helped formalize problems in a first stage, years ago, but things can now be reformulated in everyday language. The use of microcomputer programs should help to make these results more available, but a little effort should still be made by professional geologists and mining engineers to fully benefit from these results. The new direction of research into expert systems may further reduce the efforts needed, but this is still a few years down the road.

APPENDIX 1

Recovery Formulae in the Lognormal Case

The formulae to compute the recovered grade and tonnage for a given cut-off in the case of a lognormal distribution have been known for over 30 years and are repeated for instance in David (1977), and David and Dagbert (1986). Calling m the average of the distribution with no cut-off, β the logarithmic standard deviation of samples, β_v the logarithmic standard deviation of the selection unit, x_c the cut-off and $\Phi(z)$ the cumulative normal distribution, the expected grade x over the cut-off x_c is for samples:

$$x = m \frac{1 - \Phi(1/\beta\,(\mathrm{Ln}\,(x_c/m) - \beta/2)}{1 - \Phi(1/\beta\,(\mathrm{Ln}\,(x_c/m) + \beta/2)}$$

and for blocks

$$y = m \frac{1 - \Phi(1/\beta_v\,(\mathrm{Ln}\,(x_c/m) - \beta_v/2)}{1 - \Phi(1/\beta_v\,(\mathrm{Ln}\,(x_c/m) + \beta_v/2)}$$

Remembering now that $\sigma^2/m^2 = e^{\beta 2} - 1$ or $\beta^2 = \mathrm{Ln}\,(1 + \sigma^2/m^2)$, an example can be seen where

$$x_c/m = 0.5,\ \sigma/m = 1.0 \text{ and } \sigma_v/m = 0.50$$

then

$$\beta = 0.83 \text{ and } \beta_v = 0.47.$$

We obtain

$$1 - \Phi\,(1/\beta\ (\mathrm{Ln}\,(x_c/m) + \beta/2) = 1 - \Phi\,(-0.42) = 0.6528$$
$$\Phi\,(1/\beta\ (\mathrm{Ln}\,(x_c/m) - \beta/2) = 1 - \Phi\,(-1.25) = 0.895$$
$$\Phi\,(1/\beta\ (\mathrm{Ln}\,(x_c/m) + \beta_v/2) = 1 - \Phi\,(1.239) = 0.892$$
$$\Phi\,(1/\beta_v\ (\mathrm{Ln}\,(x_c/m) - \beta/2) = 1 - \Phi\,(-1.70) = 0.955$$

So that all together, the dilution on grade is

$$\frac{0.895}{0.661} - \frac{0.955}{0.892} \quad \frac{0.895}{0.662} = 21\%$$

A similar calculation gives the dilution on tons; it is 35%. All the above formulae can be programmed on a pocket calculator. A personal computer will do it of course, but it is not necessary.

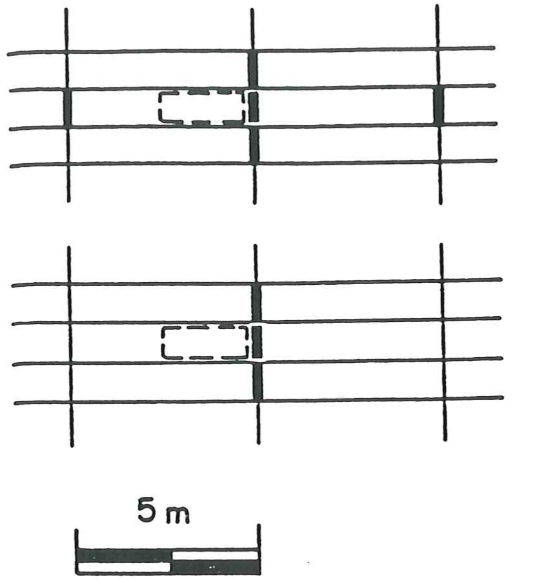

FIGURE 8a and 8b. Estimation with reduced number of samples.

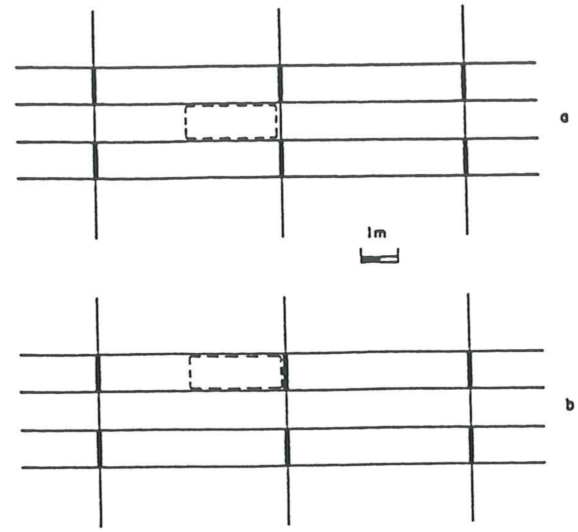

FIGURE 9a and 9b. Possible positions of a block with respect to reduced sampling pattern.

APPENDIX 2

Calculation of Block Variances

The variance of the grade of blocks in the deposits, σ^2 (V/D) is given by the difference of the variance of the grade of samples in the deposit, σ^2 (O/D) and the variance of the grade of samples within the selection unit, σ^2 (O/V):

$$\sigma^2 (V/D) = \sigma^2 (O/D) - \sigma^2 (O/V)$$

The variance of a sample in the deposit is the ordinary variance which is also equal to the sill of the variogram; the variance of a sample in the selection unit is easily obtained using charts given in David (1977) or Journel and Huijbregts (1978) or programs like KRIVAR. Examples are given in detail in David (1977).

REFERENCES

DAVID, M., 1972, Grade Tonnage Curve Use and Misuse in Ore Reserve Estimation; *Trans. Int. Min. Metall., 73 (5), pp. 175-176.*

DAVID, M., 1977, Geostatistical Ore Reserve Estimation, Elsevier, 364 p.

DAVID, M., and DAGBERT, M., 1986, Computation of Metal Recovery in Polymetallic Deposits with Selective Mining and Using Equivalent Metals; *Can. Geol. Jour. of CIM, Vol. 1, No. 1, pp. 34-37.*

DAVID, M., FROIDEVAUX, R., SINCLAIR, A.J., and VALLÉE, M., 1986, Ore Reserve Estimation Methods, Models and Reality; CIM Proceedings of the May 10-11, 1986, Symposium, 316 p.

ELBROND, J., 1986, Ore Losses, Rock Dilution and Recovery in Estimation, Design and Operation; Ore Reserve Estimation Methods, Models and Reality, CIM Proceedings of the May 10-11, 1986, Symposium, *pp. 130-134.*

Geostat Systems International Inc. (GSII), 1982, User Guide of the Geostat Package for the IBM PC.

JOURNEL, A., and HUIJBERGTS, C.J., 1978, Mining Geostatistics; Academic Press, 600 p.

KWA, B.L., and MOUSSET-JONES, P., 1986, Mineral Reserve Estimation of Gold Deposits — A Survey of Practices; *in* Ore Reserve Estimation, Methods, Models and Reality, CIM Proceedings of the May-10-11, 1986, Symposium, *pp. 172-184.*

MYERS, J.C., and SERGERIE, G., 1984, Down Loading a Mainframe Package for Reserve Modelling and Contouring; *Can. Min. Jour., Sept. 1984, pp. 17-21.*

TMS schedules productivity conference for Cologne, West Germany

"Productivity and Technology in the Metallurgical Industries" will be the theme of TMS' first ever European meeting. TMS, along with co-sponsor Gesellschaft Deutscher Metallhütten-und Bergleute, has put together a very impressive technical program featuring many distinguished international speakers. The conference is scheduled for September 17-22, 1989, at the Hyatt Regency Hotel, Cologne, West Germany.

In keeping with the conference theme, lecturers from the U.S.A., Japan, and various European countries will address the practical application of new concepts for improving productivity and technology in the primary and secondary metal industries. Leaders from the copper, nickel, lead, zinc, tin, light metals, and precious metals industries will exchange ideas on how the future challenges of new process development, energy utilization, and the market for emerging products can best be met.

Plans have been made to include a panel discussion that will address the subject of Geo-Economic Priorities. Spokesmen from North America, Europe, and the Far East will discuss the political and economic impact on technology-sensitive industrial policies.

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